(b) If $S = \begin{bmatrix} x_2^2 & x_1 x_2 \\ -x_1 x_2 & x_1^2 \end{bmatrix}$,

determine whether or not S is a Cartesian tensor of rank 2 in 2-dimension. (7)

6. Using tensors, prove the following :

(a) $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \bullet \vec{C}) - \vec{C}(\vec{A} \bullet \vec{B})$ (7)

(b) $\vec{\nabla} \times (\vec{f} \times \vec{g}) = (\vec{g} \bullet \vec{\nabla})\vec{f} - (\vec{f} \bullet \vec{\nabla})\vec{g} + \vec{f}(\vec{\nabla} \bullet \vec{g}) - \vec{g}(\vec{\nabla} \bullet \vec{f})$ (8)

7. (a) Prove that divergence of a vector field transforms like a tensor of rank 2. (5)

(b) Obtain an expression for Moment of Inertia tensor. Also, prove that it is symmetric tensor of rank 2. (10)

8. (a) A covariant tensor has components (xy, $2y - z^2$, xz). Find its covariant components in spherical co-ordinates. (10)

(b) Find g_{ij} and g corresponding to $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6\, dx^1 dx^2 + 4\, dx^2 dx^3$. (5)

[This question paper contains 4 printed pages.]

23.12.2024(M)

Your Roll No.....

Sr. No. of Question Paper : 5977 **I**

Unique Paper Code : 32227502

Name of the Paper : Advanced Mathematical Physics – I

Name of the Course : **B.Sc. (Hons.) Physics (CBCS-LOCF)**

Semester V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **All** questions carry equal marks.
4. Non-programmable Scientific calculator is allowed.

1. (a) Determine whether the set of real numbers equipped with the binary operation * defined by

$$a * b = a^b$$

forms a group or not. (5)

(b) Let T be the linear transformation on $\Re^2$ defined by

$$T(x, y) = (2x + 3y, 2x - 5y)$$

and S_1 and S_2 be two bases of $\Re^2$ given by

$$S_1 = \{(1,0), (1,1)\}, S_2 = \{(1,2), (2,1)\}$$

Find the matrix representation of T w.r.t. bases S_1 and S_2. (10)

2. (a) If H is a Hermitian matrix and I is the unit matrix of the same order as H, determine whether $A = (I - iH)(I + iH)^{-1}$ is a Unitary imatrix or not. (5)

(b) Given that eigenvalue of square matrix A is λ, prove that eigenvalue of matrix adj(A) is $\frac{|A|}{\lambda}$. (5)

(c) Find the values of x and y such that given matrix B is orthogonal :

$$B = \frac{1}{6}\begin{bmatrix} 1 & \sqrt{10} & x \\ -5 & \sqrt{10} & y \\ \sqrt{10} & 4 & \sqrt{10} \end{bmatrix} \quad (5)$$

3. (a) Find the eigenvalues and eigenvectors of the matrix C,

$$C = \frac{1}{6}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$$

Can C be diagonalised or not? Give reason(s). (10)

(b) State Cayley-Hamilton theorem. Verify it for the matrix F and hence find F^{-1}, where

$$F = \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix}. \quad (5)$$

4. (a) Find e^R for the matrix R, where

$$R = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \quad (7)$$

(b) Solve the coupled differential equations

$$\dot{y} = -y + 4y$$
$$\dot{z} = 3y - 2z$$

where, y(0) = 3, z(0) = 4 (8)

5. (a) Show that

$$\in_{ijk} \in_{kpq} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$$

and hence prove that

$$\in_{ijk} \in_{ijk} = 6 \quad (8)$$