000393

CENTRAL UNIVERSITY OF KARNATAKA

Department of Mathematics, Semester IV
End Semester Examination- April2025

Time: **150 minutes**, Subject: **Measure & Integration(Code: PMATC40015)** , Max. marks: **60**

Note: All questions are compulsory.

1. The outer measure of a bounded closed interval is its length. (6)

2. Let X be a non-empty set. Let $\mathcal{C} = \{A \subset X \mid A$ is countable, or $X \setminus A$ is countable$\}$. Then prove that $\mathcal{C}$ is a $\sigma-$algebra of subsets of X generated by $\{\{x\} \mid x \in X\}$. (6)

3. Let $E \subset \mathbb{R}$. Prove that following statements are equivalent:

    1. E is measurable.
    2. For every $\epsilon > 0$ there exists an open set O containing E with $m^*(O \setminus E) < \epsilon$.
    3. There exists a G_δ set G containing E with $m^*(G \setminus E) = 0$.

    (6)

4. Prove or disprove the following:
   Let $E_1 \supset E_2 \supset \ldots$ be measurable sets. Then $\mu(\cap_{k\in\mathbb{N}} E_k) = \lim_{k\to\infty} \mu(E_k)$. (6)

5. A real-valued function that is continuous on its measurable domain is measurable. (6)

6. State and prove Egoroff's theorem. (6)

7. Define the Lebesgue integral of a simple function. With proper justification give an example of a function that is not Riemann integrable but is Lebesgue integrable. (6)

8. Prove that a bounded measurable function is integrable over a set of finite measure. (6)

9. State and prove the Fatou's lemma. (6)

10. Prove linearity and monotonicity of integrals of general integrable functions. (6)

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Department of Mathematics, Semester-IV

End Semester Examinations-April, 2025

Time:**120 Minutes** Subject: **Theory of partitions (Code: PMATD40239)** Max mark:**45**

Note: All questions are compulsory.

1. (a) Find the general solution in terms of Gauss hypergeomtric function of

$$(x^2-1)y'' + (5x+4)y' + 4y = 0$$

near $x = 1$ and $x = -1$. (4+4)

(b) Express $sin^{-1}x$, $cos(x)$ and e^x in terms of Gauss function. (3)

2. (a) State and prove the Gauss summation formula and Chu-Vandermonde's identity. (4)

(b) State and prove the Euler pentagonal number theorem and Jacobi's identity. (4)

3. (a) State and prove the Jacobi triple product (5)

(b) State and prove the q-analogous of finite and infinite binomial theorem. (5)

4. (a) If $L(k, n, m)$ is the number of lattice paths inside the $k \times (n-k)$ rectangle, starting from (0,0) and ending at $(k, n-k)$ with the number of boxes above at most m, then find $L(2, 6, 4)$ and $L(3, 6, 3)$. (3)

(b) Give a combinatorial proof of the following identities:

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^2;q^2)_n} = (-q;q^2)_\infty,$$

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n^2} = \frac{1}{(q;q)_\infty}.$$

(4+4)

(c) Show that

$$\sum_{n=0}^{\infty} p(n)q^n = \frac{1}{(q;q)_\infty}, \quad \text{where} \quad |q| < 1.$$

(5)

000395

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School	School of Physical Sciences
Department	Department of Mathematics
Program	M.Sc. Mathematics
Semester	IV

END SEMESTER EXAMINATIONS, MONTH April YEAR 2025

Course: Calculus of Variations and Finite Element Method Code: PMATD40225	Max. Marks:45	Duration: 2 hours	Credits: 03

INSTRUCTIONS: Answer all the questions.

1. (a) Determine the extremals of the functionals $J[y(x)] = \int_{-l}^{l} (\frac{\mu y''^2}{2} + ky)dx$ subject to $y(-l) = 0, y(l) = 0, y'(-l) = 0, y'(l) = 0$.

(b) Derive the Euler's equation's for extremals of the functional. (5M+5M)

2. Find the curve passing through the points (x_1,y_1) and (x_2,y_2) such that its rotation about the x-axis give rise to a surface of revolution of minimum surface area. (5M)

3. (a) If the operator L is linear, self adjoint and positive definite then show that the necessary and sufficient condition that the solution of $L\phi = f$ in the region R with homogenous boundary conditions occurs at the minimum of the $I(\phi) = \iint_R (\phi L\phi - 2\phi f)\, dxdy$. (5M+5M)

(b) Solve $\phi'' = -x^2,\ 0 < x < 1; \phi(0) = \phi(1) = 0$ by least square method for cubic approximation.

4. (a) Derive the linear basis function for an element in two dimensional problem. (5M+5M)

(b) Show that $\iint_R L_1^m L_2^n L_3^p\, dx\, dy = 2A \frac{m!n!p!}{(m+n+p+2)!}$, where A is area of the triangular region R.

5. Solve the $\nabla^2\phi = 0,\ 0 < x, y < 1$ subject to boundary conditions $\frac{\partial\phi}{\partial x} = 0,\ x = 0, 1;\quad \phi = 1,\ y = 0$ $\frac{\partial\phi}{\partial y} + \phi = 2,\ y = 1;$ by taking two linear elements using Rayleigh Ritz method. (10M)

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School	Physical Sciences
Department	Mathematics
Program	M.Sc.
Semester	IV

END SEMESTER EXAMINATIONS, MONTH : APRIL, YEAR : 2025 (Regular)

Course: Operations Research Course Code: PMATC40016	Max. Marks: 60	Duration: 2.5 Hrs.	Credits: 04

INSTRUCTIONS: *** To answer all the following five questions is compulsory.**

*** Each question carries 12 marks.**

Answer the following.

1. A) Define a convex set. What is the importance of convex set in solution of an LPP. [2]

B)Using simplex method minimize Z = $x_1 - 3x_2 + 2x_3$,
subject to the constraint: $3x_1 - x_2 + 2x_3 \leq 7, -2x_1 + 4x_3 \leq 12, -4x_1 + 3x_2 + 8x_3 \leq 10, x_1, x_2, x_3 \geq 0$. [7]

C) Write the dual of this problem and find the values of the variables using the primal optimal table. [3]

2. A)Use penalty (or Big M) method to Maximize Z = $6x_1 + 4x_2$, subject to the constraint: [7]
$2x_1 + 3x_2 \leq 30, 3x_1 + 2x_2 \leq 24, x_1 + x_2 \geq 3, x_1, x_2, x_3 \geq 0$.
Check if this problem has more than one solution , if yes find them all.

B)If RHS of the above problem is replaced by 33, 28 and 5 units respectively, will the current basic solution be feasible? [5]

3. A)Given $x_{11} = 5, x_{12} = 10, x_{22} = 5, x_{23} = 15, x_{24} = 5. x_{34} = 10$, Is it an optimal solution to the following transportation problem. If not, find an optimal solution using modified distribution method. [7]

	1.	2.	3.	4.	Supply↓
1.	10	2	20	11	15
2.	12	7	9	20	25
3.	4	14	16	18	10
Demand→	5	15	15	15	

B) Following matrix presents four work - four job assignment cost per person per job in Rupees. Find an optimum assignment with optimum assignment cost assuming job C must not be assigned to person P and job D must not be assigned to person R [5]