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CENTRAL UNIVERSITY
OF KARNATAKA
(Established by an Act of the Parliament in 2009)
Kadaganchi, Aland Road, Kalaburagi Dist-585367
Website: www.cuk.ac.in

School	Physical Sciences
Department	Mathematics
Program	UG
Semester	I (R)
Course Code	UMATM10400

END SEMESTER EXAMINATIONS, MONTH: January, YEAR: 2024 (Repeaters)

Course: **'Differential Calculus'**	Max. Marks: **45**	Duration: **2 Hours**	Credits: **03**

INSTRUCTIONS: **1) All Questions are compulsory.**
2) All main questions carry equal marks, Figures to the right side indicate marks intended.

Q 1. **a)** State Lagrange's Mean value theorem explain geometrical interpretation of it. Verify Lagrange's Mean Value for function $f(x)=x^2-3x+2$ in [-2, 3] hence find point 'c' in given interval. [5]

b) State Rolle's theorem , what is geometrical interpretation of Rolle's theorem . Verify Rolle's theorem for $f(x) = x(x-3)^2$, on the interval [0, 3] [4]

Q 2. **a)** Let $f(x) = \frac{x^2+2}{x^2+1}$, then given $\varepsilon > 0$, find the real number $\delta > 0$ such that $|f(x)-2| < \varepsilon$ for $0 < |x| < \delta$ [3]

b) Define absolute value or modulus of a real number and prove that, $|x-y| \geq |x|-|y|$ [3]

c) Using definition of sequences show that, the sequence $\left\{ 1/3^n \right\}$ converges to 0. [3]

Q 3. **a)** Find the extreme values of the function $f(x, y) = x^3+3xy^2-3x^2-3y^2+4$ [3]

b) Expand $\log\sin(x+h)$ in the powers of 'h' by Taylor's theorem [6]

Q 4. **a)** Evaluate the limit i) $k = \lim_{x\to 0} \log_{\sin x} \sin 2x$ ii) $k = \lim_{x\to 0} \frac{x^2+2\cos x-2}{x\sin^3 x}$, applying L'Hospital's Rule. [4]

b) Explain about the continuity at a point, continuity in a closed interval also discuss the types of discontinuities.

Using the ε - ∂ definition, prove that $f(x)=\begin{cases} \frac{x^2-4}{x-2}, & if\ x \neq 2 \\ 4, & if\ x=2 \end{cases}$, is continuous at x = 2 [5]

Q 5. a) If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = \cos\theta$; show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2\sin\theta$. [6]

b) If $f(x, y) = \frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$, show that $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+2f = 0$, by using Euler's theorem. [3]

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School	School of Physical Sciences
Department	Mathematics & Computer Sciences
Program	B.Sc. MCS (CR)
Semester	I

END SEMESTER EXAMINATIONS, MONTH : JANUARY, YEAR : 2024, (Repeaters)

Course: Calculus Course Code: UMATC10100	Max. Marks: 90	Duration: 3 Hr.	Credits: 06

INSTRUCTIONS: *** Answer any six of the following questions.**

*** Each main question carries 15 marks.**

Q1) A) Check the continuity of the function f(x) defined as $f(x) = \begin{cases} x^2+2 & \text{when } x>1 \\ 2x+1 & \text{when } x=1 \\ 2 & \text{when } x<1 \end{cases}$, at x =1.

Identify the type of discontinuity , if any. [7]

B) Define derivative of a function. Show that the function $f(x) = |x|, \forall\, x \in R$ is continuous everywhere except at x=0. [8]

Q2) A) Define a homogeneous function. [10]

Z is a homogeneous function of degree n in x and y then state and prove Euler's theorem.

Hence show that $x^2\frac{\partial^2 Z}{\partial x^2} + 2xy\frac{\partial^2 Z}{\partial x \partial y} + x^2\frac{\partial^2 Z}{\partial x^2} = n(n-1)Z$.

B) Identify the order of $u = \frac{x^3+y^3}{x-y}$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$. [5]

Q3) A) Find maxima and minima of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$, if exists. [5]

B) State and prove Taylor's theorem for function of one variable. [5]

C) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find $J\left(\frac{x,y,z}{r,\theta,\phi}\right)$. [5]

Q4) A) State Rolle's theorem. Verify Rolle's mean value theorem for $(x-a)^m(x-b)^n$. [7]

B) Define radius of curvature for a curve in cartesian plane. Find radius of curvature for the curve $x^3 + y^3 = 3axy$ at the point P=(3a/2,3a/2). [8]

Q5) A) Show that $\beta(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$ [6]

B) Compute the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$. [9]

Q6) A) Trace the curve $y^2(a-x) = x^3$, a>0. [8]

B) State and prove Greens theorem. [7]

Q7) Verify Gauss- Divergence Theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, taken over the rectangular parallelopiped $0 \leq x \leq a,\ 0 \leq y \leq b, 0 \leq z \leq c$. [15]

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School	Physical Science
Department	Mathematics
Program	U.G. (B.Sc./B.A. etc...)
Semester	I Semester

END SEMESTER EXAMINATIONS, MONTH: JANUARY YEAR: 2024

Multidisciplinary Course: Matrix Theory Code: UMATD10100	Max. Marks: 45	Duration:120 Minutes	Credits: 03

INSTRUCTIONS:

1. Answer **All** questions are compulsory.

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1. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 0 & 2 \\ 4 & -3 & 2 \end{bmatrix}$ .Verify the result$(A + B)^2 = A^2 + BA + AB + B^2$. [5]

2. Using the Gauss-Jordan method, find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ [5]

3. Reduce the following matrix into its normal form and hence find its rank .

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ [5]

4. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ [5]

5. State and Prove that Cayley – Hamilton theorem. [5]

6. Define conjugate Matrix. If A= $\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$,

show that AA^* is a Hermitian matrix where A^* is the conjugate transpose of A. [5]

7. Solve by Cramer's rule

$$x - y - z = 3$$
$$2x - 6y - z = 0$$
$$-3x - 4y + 2z = 1$$ [5]

8. Explain the Symmetry properties of Matrix. [5]

9. Find matrix P which transform the matrix A to Diagonal form. Hence calculate A^4.

$$P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$ [5]

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School	Physical Science
Department	Mathematics
Program	U.G.
Semester	I Semester

END SEMESTER EXAMINATIONS, MONTH: JANUARY YEAR: 2024

Minor Course: Elementary Calculus Code: UMATM10100	Max. Marks: 90	Duration:180 Minutes	Credits: 06

INSTRUCTIONS:

1. Answer **All** questions compulsorily.

Section -A

1. A) Define limit of a sequence. Verify $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}}=2$ using definition of limit of sequence. [5]

   B) Show that the sequence $\{r^n\}$ converges iff $|r|<1$. [5]

   C) Write one divergent and one convergent sequence such that there product is [5]
   i) divergent ii) convergent iii) oscillating.

2. A) Define a homogeneous function. [8]
   Let Z be a homogeneous function of degree n in x and y then state and prove Euler's theorem. Hence verify for $u = \frac{x^3+y^3}{x-y}$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$.

   B) State and prove Rolle's Theorem. [7]

3. A) Define maxima and minima of a curve. Find maxima and minima of the function
   $f(x,y) = x^3.y^2(1-x-y)$, if exists. [6]

   B) Find Maclaurin's expansion for the function $f(x,y)= e^x\log(1+y)$. [6]

   C) If $x = r\cos\emptyset$, $y = r\sin\emptyset$, $z = z$, find $J\left(\frac{x, y, z}{r, \emptyset, z}\right)$. [3]

4. A) State Lagrange's mean value theorem. Determine a c lying between a and b for [7]
   f(x)=x(x-1)(x-2) , given a=0, b=1/2.

   B) Define double point on a curve.Trace the curve $y^2(a-x) = x^2(a+x), a > 0$. [8]

Section - B

1. Define Directional derivative. If $\frac{d\vec{a}}{dt} = \vec{w} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w} \times \vec{b}$ then show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{w} \times (\vec{a} \times \vec{b})$. [5]

2. Evaluate $\iint_A xy\,dx\,dy$, where A is the domain bounded by x-axis , ordinate $x = 2a$ and the curve $x^2 = 4ay$. [5]

3. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ [5]

4. State and Prove that Green's theorem. [5]

5. Apply Stoke's theorem to evaluate $\int_C (ydx + zdy + xdz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ $and\ x + z = a$. [5]

6. State and Prove that Gauss Divergence theorem. [5]

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School	School of physical Sciences
Department	Mathematics
Program	B.Tech (M&C)
Semester	I semester

UMACC101006

END SEMESTER EXAMINATIONS, MONTH: JANUARY, YEAR: 2024

Course: Introduction to Computing	Max. Marks: 30	Duration: 1.5 Hrs	Credits: 02

<u>INSTRUCTIONS</u>: Answer any Three questions and each question carry Ten marks.

1. (a) Write the features of C++. Explain any two user defined data types with example.
   (b) Differentiate between for, while and do-while loop.
2. (a) Define pointer. Explain any two file operations.
   Consider inserting the keys 24, 36, 58,65,62,86 into a hash table of size m=11 using linear probing.
   (b) Explain stack ADT. write C++ program for Array based implementation of stack.
3. (a) explain merge sort technique. Write C++ code for merge sort technique.
   (b) explain linear search technique. Write C++ code for linear search technique.
4. (a) Explain about preprocessor directives in C++.
   (b) Define Hashing. Explain any two open addressing methods for collision handling.

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Central University of Karnataka

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School of Physical sciences

Department of Mathematics

CENTRAL UNIVERSITY OF KARNATAKA

END SEMESTER EXAMINATION JAN-2024 SET- B

Class: I Sem

Course Name: Introduction to Electrical Engineering

Course Code:UMATC10102

Max. Marks: 45

Credits: 03

*<u>Note: Answer all the questions(All question carry equal marks:9*5=45)</u>*

1. State and explain KVL and KCL with Necessary Diagram

2. A coil having a R=12Ω , L= 0.1H is connected across a 100v 50hz supply. Calculate : Impedance , Current , Phase diff, Power Factor , Power

3. Define following terms a) Peak value b) Average value c) Active power d) Reactive power e) Apparent power

4. Explain Classification of Transformer

5. Explain the Analysis of RLC Series Circuit

6. Explain Construction and Working of Alternator

7. Fill in the blank with Appropriate Answer
   a) DC motor works on a principle of ______
   b) Which part of the Dc Generator acts as Rectifier ______
   c) Resistance offered in the AC circuits is called as _____
   d) Transformer works on the Principle of _____
   e) Give the formula for Synchronous speed of Alternator _____

8. Explain the Construction and Working of Induction Motor

9. Explain Different Types of Magnetic Material

************ALL THE VERY BEST TO YOU**************

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SCHOOL OF PHYSICAL SCIENCES
DEPARTMENT OF PHYSICS
B.Tech.(M&C)
Semester - 1 (Regular)

END SEMESTER EXAMINATIONS JANUARY 2024

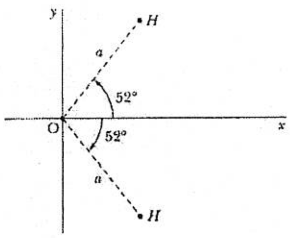
Course	Course code	Max. Marks	Duration	Credits
Engineering Physics	UMATC10101	45	2 h	3

Instructions to candidates

- Section A consists of 5 questions of 3 marks each. Answer all questions.
- Section B consists of 4 questions of 10 marks each. Answer any **THREE** questions.
- In a single question part (a) and (b) are independent.

Section A

1. Find a unit vector perpendicular to both $A = (\hat{i} + 2\hat{j} - \hat{k})$ and $B = (2\hat{i} - \hat{j} + \hat{k})$?
2. The position vector of a particle of mass 2 kg is given as a function of time by $r = (6\,\hat{i} + 5t\,\hat{j})$ m. Where t is given in seconds. Determine the angular momentum of the particle as a function of time?
3. A model of water molecule is shown below. Find out the center mass position.



4. State the Kepler's laws of planetary motion?
5. (a) What is static fluid? Briefly explain absolute, gauge, atmospheric and vacuum pressure and write the relation between them?

Section B

6. (a) State and prove the conservation of linear momentum and angular momentum of a single particle using Newtonian mechanics.
(b) Find the displacement and velocity of a particle undergoing vertical downward motion in a medium having a retarded force proportional to the velocity. $(4 + 6 = 10$ Marks$)$
7. (a) Plot and explain the stress-strain curve. $(6 + 4 = 10$ Marks$)$
(b) A wire 2 m long and 2 mm in diameter, when stretched by weight of 8 kg has its length increased by 0.24 mm. Find the stress, strain, and Young's modulus of the material of the wire. Consider 9.8 m/s^2?
8. (a) What is a central force and give one example? State the important properties of the motion under central force?
(b) Find the force law for a central force field that allows a particle to move in a spiral orbit given by $r = k\,\theta^2$, where k is constant. $(2 + 2 + 6 = 10$ Marks$)$
9. (a) Briefly explain the viscosity property of fluids? Find the coefficient of viscosity of an oil having density 981 kg/m^3. The shear stress at a point in oil is 0.2452 N/m^2 and velocity gradient at that point is 0.2 per second.
(b) State and prove Pascal's law in static fluids? $(3 + 2 + 5 = 10$ Marks$)$

—End-of-paper—

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Kadaganchi, Aland Road, Kalaburagi Dist-585367
Website: www.cuk.ac.in

School	School of Physical Sciences
Department	Mathematics
Program	B. Tech(Maths & Computing)
Semester	I

END SEMESTER EXAMINATIONS, MONTH : JANUARY, YEAR : 2024

Course: Calculus Course Code: UMATC10100	Max. Marks: 90	Duration: 3 Hr.	Credits: 06

INSTRUCTIONS: * **Each main question carries 15 marks.**

Q1) A) Define limit of a sequence. Verify $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}}=2$ using definition of limit of sequence. [6]

B) Show that the sequence $\{r^n\}$ converges iff $|r|<1$. [9]

Q2) A) Define a homogeneous function. [10]

Let Z be a homogeneous function of degree n in x and y then state and prove Euler's theorem. Hence show that $x^2\frac{\partial^2 Z}{\partial x^2}+2xy\frac{\partial^2 Z}{\partial x\partial y}+x^2\frac{\partial^2 Z}{\partial x^2}=n(n-1)Z$.

B) Identify the order of $u=\frac{x^3+y^3}{x-y}$. Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$. [5]

Q3) A) Find maxima and minima of the function $f(x,y)=x^3+y^3-3x-12y+20$, if exists. [5]

B) Find Maclaurin's expansion for the function $f(x,y)= e^x\log(1+y)$ [5]

C) If $x = r\cos\emptyset\sin\theta$, $y=r\sin\theta\sin\emptyset$, $z=r\cos\theta$ find $J\left(\frac{x,y,z}{r,\theta,\emptyset}\right)$.

Q4) A) State Rolle's theorem. Verify Rolle's mean value theorem for $(x-a)^m(x-b)^n$ [5]

B) Define double point on a curve.Trace the curve $y^2(a-x)=x^2(a+x), a>0$. [10]

Q5) A) Show that $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$, Hence evaluate $\Gamma(1/2)$. [6]

B) Express the following as Beta function $\int_0^2 \sqrt{x}\,(4-x^2)^{-1/4}dx$. [4]

C) Find constants a,b,c if the vector [5]

$\bar{f}=(2x+3y+az)\bar{i}+(bx+2y+3z)\bar{j}+(2x+cy+3z)\bar{k}$ is irrotational.

Q6) A) Change the order of integration and evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$. [5]

B) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dxdydz.$ [5]

C) Given $\overline{F} = 3xy\overline{i} - y^2\overline{j}$, evaluate $\int_c \overline{F}.\overline{dr}$, where c is the curve $y = 2x^2$ in the xy-plane [5]

from (0,0) to (1,2).

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CENTRAL UNIVERSITY OF KARNATAKA, KALABURAGI-585367

Department of Mathematics, School of Physical Sciences

End Semester Examinations-January 2024

Subject: Introduction to Linear Algebra (Code: UMATM30200)

Time: **150 minutes** Minor Course (For III Sem., B.Sc. Computer Sciences) Max. marks: **60**

Note: All questions are compulsory.

1. (a) Find k so that $u = (1, 3, 4, k)$ and $v = (2, 4, 5, 6)$ are orthogonal in $\mathbb{R}^4$. (1)

   (b) Define the Rank-Nullity theorem. (1)

   (c) Is $\mathbb{R}$ a vector space over a field $\mathbb{C}$? Justify your answer. (1)

   (d) Define an orthogonal basis and orthonormal basis of an inner product space V. (1)

   (e) Does the vectors $u = (-3, 7)$ and $v = (5, 5)$ form a basis for $\mathbb{R}^2$? (1)

   (f) Find the characteristic polynomial of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. (1)

   (g) Is $W = \{(x, y, z) \in \mathbb{R}^3 / x + 2y + z \geq 0, x, y, z \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$? Explain (1)

   (h) Consider the vector space of real polynomials $P(t)$ with the inner product

   $$\langle f, g \rangle = \int_0^1 f(t)g(t)dt.$$

   Find $\langle f, g \rangle$ and $\langle f, h \rangle$, where $f(t) = t$, $g(t) = 3t$ and $h(t) = t^2 - 2t$. (1)

   (i) Define inner product space. (1)

   (j) Is $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ an orthonormal basis of $M_2(\mathbb{R})$, where $\langle A, B \rangle = trac(B^t A), \forall A, B \in M_2(\mathbb{R})$?. Explain (1)

2. (a) Find the angle between $u = (1, 3, 0, 2)$ and $v = (2, -3, 4, 1)$ in $\mathbb{R}^4$. (2)

   (b) Define the basis and dimension of a vector space. (2)

   (c) If 7 and -12 are eigenvalues of a matrix $A_{3\times3}$, then find the determinate and trace of A. (2)

   (d) Show that the vectors $S = \{(1, 4, 7), (2, 5, 8), (3, 6, 9)\}$ are linearly dependent in $\mathbb{R}^3$. (2)

   (e) Is $S = \{(1, 1, 0), (1, 0, 1), (0, 1, -1)\}$ a basis of $\mathbb{R}^3$? Explain. (2)

3. (a) Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix representation of T relative to the basis $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. (4)

   (b) Find the eigenvalues and minimal polynomial of the matrix $\begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & -1 \\ -1 & 1 & 1 \end{bmatrix}$. (6)

4. (a) Find the basis and dimension of the subspace W of $M_2(\mathbb{R})$ spanned by $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$. (5)

   (b) Find the coordinate vector of $7 - t + 2t^2$ relative to the basis $S = \{1 + t + t^2, t + t^2, t^2\}$ for vector space of polynomials of degree ≤ 2 over a field $\mathbb{R}$. (5)

5. Let V be the vector space of polynomials of at most degree 3 with inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$. Apply Gram-Schmidt orthogonalization process to $S = \{1, t, t^2, t^3\}$ to find an orthogonal basis and then an orthonormal basis. (10)

6. Let V be a vector space over a field F and let $x, y \in V$.Then prove that

(a) $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$ (5)

(b) $\|x + y\| \leq \|x\| + \|y\|$ (5)